# Impressions of R from an Introductory Course on Statistics 

William John Holden

10 May 2020

```
library(readxl)
library(ggplot2)
library(dplyr)
library(tidyr)
library(comprehenr)
library(knitr)
```


## Introduction

Over the past eight weeks I took Introduction to Statistics (STAT 200) at the University of Maryland Global Campus (UMGC). My bachelor's program in computer science did not require a course in statistics and I felt this was something I needed before applying to graduate school.
UMGC teaches STAT200 from the second edition of Statistics Using Technology by Kathryn Kozak. UMGC does not require the use of any specific technology for this course. The textbook shows functions from a Texas Instruments calculator. You could probably get away with Microsoft Excel if you are very clever. I took this opportunity to learn R.
I would rate my own skill in computer programming somewhere above novice but below intermediate. My day job is in computer networking, where I do not write code every day. Most of my programs are in (in descending order of strength) Java, Julia, JavaScript, and Mathematica.

This document is a summary of my experience self-teaching R. I did not bother trying to learn $R$ as a general-purpose programming language and I have no intention to use R for Advent of Code. I think of R as a domain-specific language for doing statistics. I would never consider R for building a web application, video game, desktop application, or network service.

## RStudio

I used a program called RStudio. There is a no-cost, open-source version of RStudio Desktop. Before you install RStudio you will need the core R language, which you can download from CRAN.

RStudio is great. The core R language is a REPL. RStudio provides both a code view and console view that allow you to effortlessly switch from REPL to code. RStudio's graphical user interface provides view of your environment variables, which feels a little bit like a debugger in a standard IDE. RStudio prominently features a view for plots, which makes sense for a product aimed at statisticians.

## R Markdown

R Markdown is a "killer app." R Markdown is a notebook interface where you mix prose, mathematical notation, code, and code output. RStudio can output to LaTeX. If you already know LaTeX then you can less R Markdown in about 2 minutes (which includes the 60 seconds to learn Markdown.

## Cosma Shalizi has an introduction to R Markdown at Using R Markdown for Class Reports.

The other two notebook interfaces I have used are Mathematica and Jupyter. Mathematical notation is important to me. I hated the palettes in Mathematica. I never memorized the keyboard shortcuts and the graphical user interface was painfully slow and difficult to use. The equation editor in Microsoft Office is easier to use. I had no trouble learning Jupyter because I had already invested in learning LaTeX. I recent watched an excellent talk by Joel Grus called "I don't like notebooks". Grus' chief frustration with notebooks is hidden and confusing state, such as out-of-order evaluation. I think RStudio users might be less likely to encouter these problems than Jupyter users. An RStudio user is likely to try their computation at the console first, then save their history to an R Script when they are satisfied with their result. Using the environment as a debugger to look for a missing step should be a natural first step if the R Script does not work as expected.

R Markdown manages to put together a lot of things that worked in the industry. Prose, code, and plots are easily blended, just like Mathematica Jupyter notebooks. The source document is plain text and is readable in this form. Finally, you can use LaTeX to typeset beautiful math.

## Reading Data

RStudio has a very useful Import Dataset feature. You can use this feature to navigate to a file, such as an Excel spreadsheet, containing your data, view the columns. The feature shows the R source code needed to produce the result you see on the screen. readxls is slick. For example:

```
library(readxl)
CrossFit <- read_excel("CrossFit.xlsx")
kable(CrossFit)
```

| Study.Authors | Year | x.Male | x.Female | x | n.Male | n.Female | n | Incidence |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Hak, Hodzovic, \& Hickey | 2013 | NA | NA | 97 | NA | NA | 132 | 3.10 |
| Giordano \& Weisenthal | 2014 | NA | NA | NA | NA | NA | 386 | 2.40 |
| Weisenthal et al. | 2014 | 53 | 21 | 75 | 231 | 150 | 386 | NA |
| Chachula, Cameron, \& Svoboda | 2015 | NA | NA | 24 | 40 | 14 | 54 | NA |
| Aune \& Powers | 2016 | 52 | 33 | 85 | 142 | 105 | 247 | 2.71 |
| Sprey et al. | 2016 | 109 | 67 | 176 | 323 | 243 | 566 | NA |
| Summitt et al. | 2016 | NA | NA | 44 | NA | NA | 187 | 1.94 |
| Mehrab et al. | 2017 | 157 | 95 | 252 | 266 | 183 | 449 | NA |
| Montalvo et al. | 2017 | 30 | 20 | 50 | 94 | 97 | 191 | 2.30 |
| Moran et al. | 2017 | NA | NA | NA | 66 | 51 | 117 | 2.10 |
| Feito, Burrows, \& Tabb | 2018 | 495 | 436 | 931 | 1566 | 1483 | 3049 | 0.74 |
| Tafuri et al. | 2019 | NA | NA | 181 | 325 | 129 | 454 | NA |
| da Costa et al. | 2019 | 89 | 68 | 157 | 243 | 171 | 414 | 3.24 |
| Minghelli \& Vicente | 2019 | NA | NA | 61 | 152 | 118 | 270 | 1.34 |
| Alekseyev et al. | 2020 | 198 | 97 | 295 | 589 | 296 | 885 | NA |
| Larsen et al. | 2020 | NA | NA | 25 | 51 | 117 | 168 | 10.60 |
| Szeles et al. | 2020 | NA | NA | NA | 198 | 208 | 406 | 18.90 |

The kable function helps format a data frame into a format that looks nice in LaTeX. In RStudio you can call View (with capital "V") or enter the name of your variable into the console and press enter.

## Tidyverse

Getting data into any computing environment can be difficult. During Advent of Code 2019 I felt Julia made importing plaintext input fairly easy with its readdlm. I had a less positive experience of Mathematica's Import command in Advent of Code 2018. Granted, I did not use R for general-purpose computing.

Getting data into a usable format for R can be difficult and tedious. I recommend watching this four-video series called "Data Wrangling with R and the Tidyverse" by Garrett Grolemund:

1. Introduction to Data Wrangling
2. Tidy Data and tidyr
3. Data Manipulation Tools: dplyr
4. Working with Two Datasets: Binds, Set Operations, and Joins

Tidyverse refers to to a collection of R packages that work well together to manipulate and visualize data. The two packages that I mainly used in STAT200 were dplyr and ggplot2.

## $\%>\%$

One of my favorite features in the Tidyverse is the $\%>\%$ operator. This operator pipes input from the last command as an anonymous variable and supplies this anonymous variable as the first parameter of the next function. So, $g(f(x, y), z)$ is semantically the same as $f(x, y) \%>\% g(z)$. This may look strange but it very easy to use in practice.

I really like pipelines in bash and Powershell. Let's compare the sum of the square roots of five random numbers using a pipeline in a few programming languages.

$$
\sum_{i=1}^{5} \sqrt{\operatorname{Random}[0,1]}
$$

Java 8's Stream API can do this in a predictably verbose form.
DoubleStream.generate(Math::random).limit(5).map(Math::sqrt).sum()
JavaScript truly is a Lisp in C's clothing.

```
Array.from({length: 5}, Math.random).map(Math.sqrt).reduce((a,b) => a + b, 0)
```

Julia has a weird $\mid>$ operator, but you have to explicitly state the name of variables passed to some functions. All of these do the same thing:

```
rand(5) |> x -> map(sqrt, x) |> sum
rand(5) |> x -> sqrt.(x) |> sum
sum(map(sqrt, rand(5)))
sum(sqrt.(rand(5)))
```

Julia's sqrt function is not vectorized. sqrt expects a single value; you can map or the dot syntax to apply a function element-wise against an array.

Here is the R version:
sum(sqrt(runif(5)))
\#\# [1] 3.198304
runif(5) \%>\% sqrt() $\%>\%$ sum()
\#\# [1] 3.064623

## Vectorized Functions

The sqrt function in $R$ is built on an assumption that, if given a list, it should operate element-wise and return a list of the same length. Many of R's built-in functions are vectorized:

```
1:10 + 0
```

\#\# [1] $1 \begin{array}{llllllllll} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 10\end{array}$

```
1:10 * runif(10)
```

\#\# [1] 0.9666563 1.25496972 .27375682 .19377302 .53788711 .46702443 .9905612
\#\# [8] 6.8578826 6.7414081 3.1413934
$\exp (1: 10) / 2^{\sim}(1: 10)$

| \#\# | $[1]$ | 1.359141 | 1.847264 | 2.510692 | 3.412384 | 4.637911 | 6.303575 | 8.567447 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| \#\# | $[8]$ | 11.644367 | 15.826336 | 21.510221 |  |  |  |  |

If one vector is longer than the other then R will wrap around and reuse values from the shorter vector. This is completely different from Python's Python's zip function.

```
1:10 + 1:9 + 1:8
## Warning in 1:10 + 1:9: longer object length is not a multiple of shorter object
## length
## Warning in 1:10 + 1:9 + 1:8: longer object length is not a multiple of shorter
## object length
## [1] [llllllllllll
```


## dplyr

Some of the functions that I found especially useful in dplyr are arrange, filter, select, mutate, and summarize. There are many more powerful and highly composable functions in dplyr, such as pivot_longer (gather) and pivot_wider (spread), but I did not use them as often.

## arrange

Sort the studies in descending order by year, then alphabetically by the authors' names.
CrossFit \% \% \% arrange(-Year, Study.Authors) \% \% \% kable()

| Study.Authors | Year | x.Male | x.Female | x | n.Male | n.Female | n | Incidence |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Alekseyev et al. | 2020 | 198 | 97 | 295 | 589 | 296 | 885 | NA |
| Larsen et al. | 2020 | NA | NA | 25 | 51 | 117 | 168 | 10.60 |
| Szeles et al. | 2020 | NA | NA | NA | 198 | 208 | 406 | 18.90 |
| da Costa et al. | 2019 | 89 | 68 | 157 | 243 | 171 | 414 | 3.24 |
| Minghelli \& Vicente | 2019 | NA | NA | 61 | 152 | 118 | 270 | 1.34 |
| Tafuri et al. | 2019 | NA | NA | 181 | 325 | 129 | 454 | NA |
| Feito, Burrows, \& Tabb | 2018 | 495 | 436 | 931 | 1566 | 1483 | 3049 | 0.74 |
| Mehrab et al. | 2017 | 157 | 95 | 252 | 266 | 183 | 449 | NA |
| Montalvo et al. | 2017 | 30 | 20 | 50 | 94 | 97 | 191 | 2.30 |
| Moran et al. | 2017 | NA | NA | NA | 66 | 51 | 117 | 2.10 |
| Aune \& Powers | 2016 | 52 | 33 | 85 | 142 | 105 | 247 | 2.71 |
| Sprey et al. | 2016 | 109 | 67 | 176 | 323 | 243 | 566 | NA |
| Summitt et al. | 2016 | NA | NA | 44 | NA | NA | 187 | 1.94 |
| Chachula, Cameron, \& Svoboda | 2015 | NA | NA | 24 | 40 | 14 | 54 | NA |
| Giordano \& Weisenthal | 2014 | NA | NA | NA | NA | NA | 386 | 2.40 |
| Weisenthal et al. | 2014 | 53 | 21 | 75 | 231 | 150 | 386 | NA |
| Hak, Hodzovic, \& Hickey | 2013 | NA | NA | 97 | NA | NA | 132 | 3.10 |

## filter and select

Show only the studies which reported injury incidence (injuries per 1000 hours of exposure), and show only this column and the authors.

```
CrossFit %>% filter(!is.na(Incidence)) %>% select(Study.Authors,Incidence) %>% kable()
```

| Study.Authors | Incidence |
| :--- | ---: |
| Hak, Hodzovic, \& Hickey | 3.10 |
| Giordano \& Weisenthal | 2.40 |
| Aune \& Powers | 2.71 |
| Summitt et al. | 1.94 |
| Montalvo et al. | 2.30 |
| Moran et al. | 2.10 |
| Feito, Burrows, \& Tabb | 0.74 |
| da Costa et al. | 3.24 |
| Minghelli \& Vicente | 1.34 |
| Larsen et al. | 10.60 |
| Szeles et al. | 18.90 |

In this situation, you could get the same result with the drop_na function from tidyr:

```
CrossFit %>% select(Study.Authors, Incidence) %%% drop_na()
```


## mutate

Compute injury prevalence (the proportion of athletes injured in a sample) by gender:

```
CrossFit <- CrossFit %>% mutate(p.Male = x.Male / n.Male, p.Female = x.Female / n.Female,
    p = x / n)
CrossFit %>% select(Study.Authors, Year, p.Male, p.Female, p) %>% kable()
```

| Study.Authors | Year | p.Male | p.Female | p |
| :--- | ---: | ---: | ---: | ---: |
| Hak, Hodzovic, \& Hickey | 2013 | NA | NA | 0.7348485 |
| Giordano \& Weisenthal | 2014 | NA | NA | NA |
| Weisenthal et al. | 2014 | 0.2294372 | 0.1400000 | 0.1943005 |
| Chachula, Cameron, \& Svoboda | 2015 | NA | NA | 0.4444444 |
| Aune \& Powers | 2016 | 0.3661972 | 0.3142857 | 0.3441296 |
| Sprey et al. | 2016 | 0.3374613 | 0.2757202 | 0.3109541 |
| Summitt et al. | 2016 | NA | NA | 0.2352941 |
| Mehrab et al. | 2017 | 0.5902256 | 0.5191257 | 0.5612472 |
| Montalvo et al. | 2017 | 0.3191489 | 0.2061856 | 0.2617801 |
| Moran et al. | 2017 | NA | NA | NA |
| Feito, Burrows, \& Tabb | 2018 | 0.3160920 | 0.2939987 | 0.3053460 |
| Tafuri et al. | 2019 | NA | NA | 0.3986784 |
| da Costa et al. | 2019 | 0.3662551 | 0.3976608 | 0.3792271 |
| Minghelli \& Vicente | 2019 | NA | NA | 0.2259259 |
| Alekseyev et al. | 2020 | 0.3361630 | 0.3277027 | 0.3333333 |
| Larsen et al. | 2020 | NA | NA | 0.1488095 |
| Szeles et al. | 2020 | NA | NA | NA |

As shown, $R$ handles missing values gracefully with the special constant and reserved keyword NA.

## summarize

Find the sum of two columns. Ignore NA values in the $x$ column.

```
CrossFit %>% summarise(x.total = sum(x, na.rm = TRUE), n.total = sum(n)) %>%
    mutate(p.total = x.total / n.total) %>% kable()
```

| x.total | n.total | p.total |
| ---: | ---: | ---: |
| 2453 | 8361 | 0.293386 |

## ggplot2

ggplot2 is a powerful tool for creating beautiful plots. It is not trivial to learn, but you get away with searching the Internet for what you need. 'ggplot2 can accept input from the $\%>\%$ operator, but you use the + operator to compose ggplot2 functions. ggplot2 can produce pie charts, but apparently pie charts have fallen out of favor. Here are some simple examples:

## Box Plot

```
CrossFit %>% ggplot(mapping = aes(x = n)) + geom_boxplot()
```



## Histogram

```
CrossFit %>% ggplot(aes(Incidence)) + geom_histogram(binwidth = 1)
## Warning: Removed 6 rows containing non-finite values (stat_bin).
```



## Points

```
CrossFit \%>\% ggplot(aes(x = Year)) + geom_point(aes(y = p.Male, color="Male")) +
    geom_point(aes(y = p.Female, color="Female"))
\#\# Warning: Removed 9 rows containing missing values (geom_point).
\#\# Warning: Removed 9 rows containing missing values (geom_point).
```



## Basic Plots

Vanilla R also has basic plotting utilities, such as pie, hist, plot, and boxplot. These functions have a little bit less functionality than ggplot2 and (at least by default) and less pretty, but the commands are simpler and easier to figure out. The basic functions are actually more difficult to use when you want something that isn't trivial. For example, compare a

Mathematica's Show command is a little easier to use. In R, it is not completely obvious how to show multiple series on a single plot. Here is one way:

```
# Base R
Injuries <- CrossFit %>% select(starts_with("x"), starts_with("n")) %>% drop_na()
plot(Injuries$n.Male, Injuries$x.Male, xlab = "Sample Size", ylab = "Injuries",
    col = "blue", xlim = c(0,1600), ylim = c(0,500))
par(new = TRUE)
plot(Injuries$n.Female, Injuries$x.Female, xlab = "", ylab = "", col = "red",
    axes = FALSE)
abline(lm(Injuries$x.Male ~ Injuries$n.Male), col = "blue")
abline(lm(Injuries$x.Female ~ Injuries$n.Female), col = "red")
```



```
# ggplot2, arguably simpler for non-trivial plots.
Injuries %>% ggplot() + geom_point(aes(n.Male, x.Male, color = "Males")) +
    geom_smooth(method = lm, formula = y ~ x, aes(n.Male, x.Male, color = "Males"),
        se = FALSE) +
    geom_point(aes(n.Female, x.Female, color = "Females")) +
    geom_smooth(method = lm, formula = y ~ x, aes(n.Female, x.Female, color = "Females"),
        se = FALSE) +
    xlab("Sample Size") +
    ylab("Injuries") +
    labs(color = "Gender")
```



## R internals

If ever you need to negate any "restored my faith in humanity" clickbait then have a look in R's source code. Here is pnorm.c. Now I appreciate why it is an advantage for much of Julia to be itself implemented in Julia.

## List Comprehension

I didn't initially appreciate list comprehension in Python when I first encountered it. Now I love it. Mathematica has a similar concept in its Array function and Julia provides the same syntax as Python. There is a comprehenr library that gives R the same functionality.

```
fibonacci <- function(n) {
    if (n == 0 | n == 1)
        return(1)
    else
        return(fibonacci(n-1) + fibonacci(n-2))
}
to_vec(for(i in 0:20) fibonacci(i))
\begin{tabular}{rrrrrrrrrrrrr} 
\#\# [1] & 1 & 1 & 2 & 3 & 5 & 8 & 13 & 21 & 34 & 55 & 89 & 144 \\
\#\# & {\([13]\)} & 233 & 377 & 610 & 987 & 1597 & 2584 & 4181 & 6765 & 10946 & & \\
\end{tabular}
```


## Basic Statistics

Ok, so I have talked about R and compared it to a few other programming languages that I am comfortable with. The remainder of this document is basically my notes for my future self to refer to once I have forgotten everything.

## Basic Stats

```
WorldSeries <- data.frame(Year = 1903:2019,
    Length = c(8, NA, 5, 6, 5, 5, 7, 5, 6, 8, 5, 4, 5, 5, 6, 6, 8,
    7, 8, 5, 6, 7, 7, 7, 4, 4, 5, 6, 7, 4, 5, 7, 6, 6, 5,
```

$4,4,7,5,5,5,6,7,7,7,6,5,4,6,7,6,4,7$, $7,7,7,6,7,5,7,4,7,7,4,7,7,5,5,7,7,7$, $5,7,4,6,6,7,6,6,7,5,5,7,7,7,5,4,4,7$, $6,6, N A, 6,6,7,4,4,5,7,7,6,4,4,5,4,5$, $6,5,7,4,6,7,5,7,7,5,7)$ )
\# see also mean(), median(), and range()
summary (WorldSeries)

| \#\# | Year | Length |
| :---: | :---: | :---: |
| \#\# | Min. :1903 | Min. $: 4.000$ |
| \#\# | 1st Qu.:1932 | 1st Qu.:5.000 |
| \#\# | Median :1961 | Median :6.000 |
| \#\# | Mean :1961 | Mean :5.843 |
|  | 3rd Qu.:1990 | 3rd Qu.:7.000 |
| \#\# | Max. :2019 | Max. :8.000 |
| \#\# |  | NA's :2 |

\# variance and sample standard deviation
var(WorldSeries\$Length, na.rm = TRUE)
\#\# [1] 1.378795
sd(WorldSeries\$Length, na.rm = TRUE)
\#\# [1] 1.174221
\# the hist() function returns an object with many useful fields.
h <- hist(WorldSeries\$Length, breaks = 4:9, right = FALSE)

Histogram of WorldSeries\$Length

data.frame(Length $=(h \$ b r e a k s)[-l e n g t h(h \$ b r e a k s)]$,
Frequency = h\$counts, Relative = h\$density,
Cumulative = cumsum(h\$density)) \% $>\%$ kable()

| Length | Frequency | Relative | Cumulative |
| ---: | ---: | ---: | ---: |
| 4 | 19 | 0.1652174 | 0.1652174 |
| 5 | 28 | 0.2434783 | 0.4086957 |


| Length | Frequency | Relative | Cumulative |
| ---: | ---: | ---: | ---: |
| 6 | 24 | 0.2086957 | 0.6173913 |
| 7 | 40 | 0.3478261 | 0.9652174 |
| 8 | 4 | 0.0347826 | 1.0000000 |

## Discrete Probability

The Binomal Formula for the probability of $r$ successes in $n$ trials is

$$
P(x=r)=\binom{n}{r} p^{r} q^{n-r}
$$

```
# combination
for (i in 0:8) {
    print(choose(i, 0:i))
}
```

\#\# [1] 1
\#\# [1] 11
\#\# [1] 121
\#\# [1] 1331
\#\# [1] 14641
\#\# [1] $1 \begin{array}{llllll}5 & 10 & 10 & 5 & 1\end{array}$
\#\# [1] $1 \begin{array}{lllllll}6 & 15 & 20 & 15 & 6 & 1\end{array}$
\#\# [1] $1 \begin{array}{lllllllll} & 7 & 21 & 35 & 35 & 21 & 7 & 1\end{array}$

\# factorial
factorial(0:8)
\#\# [1] $1 \begin{array}{llllllllll} & 1 & 1 & 2 & 6 & 24 & 120 & 720 & 5040 & 40320\end{array}$
\# probability of getting EXACTLY 0-10 heads of 10 fair die rolls.
dbinom(x = 0:10, size $=10$, prob = 1/6)
\#\# [1] 1.615056e-01 3.230112e-01 $2.907100 \mathrm{e}-01 \quad 1.550454 \mathrm{e}-01 \quad 5.426588 \mathrm{e}-02$
\#\# [6] 1.302381e-02 2.170635e-03 2.480726e-04 1.860544e-05 8.269086e-07
\#\# [11] 1.653817e-08
sum(dbinom(x = 0:10, size = 10, prob = 1/6))
\#\# [1] 1
\# cumulative probability of getting AT LEAST 0-10 heads.
cumsum(dbinom(x = 0:10, size $=10$, prob $=1 / 6$ ))
\#\# [1] $0.16150560 .4845167 \quad 0.7752268 \quad 0.93027220 .9845380 \quad 0.99756180 .9997325$
\#\# [8] 0.9999806 0.99999921 .00000001 .0000000
pbinom( $q=0: 10$, size $=10$, prob $=1 / 6$ )
\#\# [1] 0.1615056 $0.48451670 .7752268 \quad 0.93027220 .98453800 .99756180 .9997325$
\#\# [8] 0.99998060 .99999921 .00000001 .0000000
\# probability of getting MORE THAN 0-10 heads.
pbinom(q = 0:10, size = 10, prob = 1/6, lower.tail = FALSE)

```
## [1] 8.384944e-01 5.154833e-01 2.247732e-01 6.972784e-02 1.546197e-02
## [6] 2.438156e-03 2.675215e-04 1.944889e-05 8.434468e-07 1.653817e-08
## [11] 0.000000e+00
1 - pbinom(q = 0:10, size = 10, prob = 1/6)
## [1] 8.384944e-01 5.154833e-01 2.247732e-01 6.972784e-02 1.546197e-02
## [6] 2.438156e-03 2.675215e-04 1.944889e-05 8.434468e-07 1.653817e-08
## [11] 0.000000e+00
# inverse of cumulative distribution function:
# how many heads are in p% of 10 fair rolls?
qbinom(p = seq(0, 1, .1), size = 10, prob = 1/6)
```

\#\# [1] 00

In a binomial distribution $\mu=n p$ and $\sigma^{2}=n q p$ where $q=1-p$.

## Continuous Probability

The normal distribution's probability density function is

$$
\frac{1}{\sigma \sqrt{2 \pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}} .
$$

A $z$-score is computed from

$$
z=\frac{x-\mu}{\sigma} .
$$

\# probability density function for a normal distribution of default mean $=0$ and sd $=1$. plot(dnorm, xlim $=c(-5,5)$ )


```
# cumulative probability
plot(pnorm, xlim = c(-5,5))
```


\# inverse of cumulative probability plot(qnorm)


```
# Empirical Rule 68% of data lies within +/-1sd, 95% +/-2sd, 99.7% +/-3sd.
1 - 2 * pnorm(-1:-3)
## [1] 0.6826895 0.9544997 0.9973002
# if the average of some normal distribution is 100 and standard deviation is 20,
# what is the probability of a quantity between 60 and 90?
pnorm(q = 90, mean = 100, sd = 20) - pnorm(q = 60, mean = 100, sd = 20)
## [1] 0.2857874
integrate(function(x) dnorm(x, mean = 100, sd = 20), lower = 60, upper = 90)
## 0.2857874 with absolute error < 3.2e-15
```

```
# normal probability plot. This was not really emphasized in my textbook. It is a way to
# assess if your data forms a normal distribution.
qqnorm(c(CrossFit$p.Male, CrossFit$p.Female))
```


## Normal Q-Q Plot



Central Limit Theorem: suppose a random variable is from any distribution. If a sample of size $n$ is taken, then the sample mean, $\bar{x}$, becomes normally distributed as $n$ increases.

## The rest

The remainder of this paper is an extremely quick review of some important statistical functions in R. I have run out of time and cannot make this review as comprehensive as I would like. Forgive me, future self!

I have run out of time to write these notes. So far I have only covered the first half of the class. The remainder of the class covered the 1-prop test, 1 sample test for the mean (t-Test), 1-prop interval, t-interval, 2-prop test, 2-sample paired t-Test, 2-sample t-Test, 2-sample t-interval, regression, $r^{2}, \chi^{2}$ test, and ANOVA test.

Here are the short descriptions.

## $t$-Test

The $t$ test allows you to reason about small samples. R contains the functions dt , pt , and qt that largely work the same way as dnorm, pnorm, and qnorm. The major difference is the degrees of freedom. Usually, $d f=n-1$. You also have the wonderful $t . t e s t$ (which can accept a paired parameter) which does everything for you.
t.test can help you answer whether $\mu_{1}=\mu_{2}$ and estimate the difference in means,

$$
\left(\bar{x}_{1}-\bar{x}_{2}\right)-E<\mu_{1}-\mu_{2}<\left(\bar{x}_{1}-\bar{x}_{2}\right)+E .
$$

```
t.test(CrossFit$p.Male, CrossFit$p.Female)
##
## Welch Two Sample t-test
##
## data: CrossFit$p.Male and CrossFit$p.Female
## t = 0.88263, df = 13.839, p-value = 0.3925
```

```
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -0.0691794 0.1657546
## sample estimates:
## mean of x mean of y
## 0.3576225 0.3093349
# we get a completely different result when we pair the male/female statistics by study.
t.test(CrossFit$p.Male, CrossFit$p.Female, paired = TRUE)
##
## Paired t-test
##
## data: CrossFit$p.Male and CrossFit$p.Female
## t = 2.9296, df = 7, p-value = 0.02204
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## 0.009312098 0.087263152
## sample estimates:
## mean of the differences
## 0.04828762
```


## Proportion test

prop.test lets you compare proportions of "successes" and determine if they are the same.

```
# unsurprisingly, some studies are have MUCH higher proportions than others
```

prop.test(CrossFit\$x.Male, CrossFit\$n.Male)
\#\#
\#\# 8-sample test for equality of proportions without continuity
\#\# correction
\#\#
\#\# data: CrossFit\$x.Male out of CrossFit\$n.Male
\#\# X-squared $=91.782$, $d f=7, p-v a l u e<2.2 e-16$
\#\# alternative hypothesis: two.sided
\#\# sample estimates:
\#\# prop 3 prop 5 prop 6 prop 8 prop 9 prop 11 prop 13 prop 15
\#\# 0.22943720 .36619720 .33746130 .59022560 .31914890 .31609200 .36625510 .3361630

You can also use prop.test to find confidence intervals.

```
# confidence interval from third study
prop.test(CrossFit$x[3], CrossFit$n[3], conf.level = 0.90)
##
## 1-sample proportions test with continuity correction
##
## data: CrossFit$x[3] out of CrossFit$n[3], null probability 0.5
## X-squared = 143.07, df = 1, p-value < 2.2e-16
## alternative hypothesis: true p is not equal to 0.5
## 90 percent confidence interval:
## 0.1621469 0.2308752
## sample estimates:
##
    p
## 0.1943005
```

This means that, from the data from the third study, there is a $90 \%$ probability that the parameter proportion of athletes injured in CrossFit is on the calculated interval.

## Regression

$R$ makes linear regressions really easy.

```
model <- lm(CrossFit$x.Male ~ CrossFit$n.Male)
summary(model)
##
## Call:
## lm(formula = CrossFit$x.Male ~ CrossFit$n.Male)
##
## Residuals:
\#\# Min 1Q Median 3Q Max
## -32.831 -8.115 -4.347 -0.023 60.352
##
## Coefficients:
## Estimate Std. Error t value Pr(>|t|)
## (Intercept) 14.43716 14.11245 1.023 0.346
## CrossFit$n.Male 0.30906 0.02261 13.667 9.53e-06 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 28.82 on 6 degrees of freedom
## (9 observations deleted due to missingness)
## Multiple R-squared: 0.9689, Adjusted R-squared: 0.9637
## F-statistic: 186.8 on 1 and 6 DF, p-value: 9.531e-06
```


## $\chi^{2}$ Test

The $\chi^{2}$ test can tell if two variables are dependent using the equation

$$
\chi^{2}=\sum \frac{(O-E)^{2}}{E}
$$

where $O$ is the observed frequency and $E$ is the expected frequency.

```
# Is injury prevalence equal across all studies? Probably not.
chisq.test(x = CrossFit$x, y = CrossFit$n)
## Warning in chisq.test(x = CrossFit$x, y = CrossFit$n): Chi-squared approximation
## may be incorrect
##
## Pearson's Chi-squared test
##
## data: CrossFit$x and CrossFit$n
## X-squared = 182, df = 169, p-value = 0.234
CrossFit %>% select(x, n) %>% drop_na() %>% ggplot(aes(n, x)) + geom_point()
```



## ANOVA

What is the probability that the mean engine size is the same for all number of cylinders?

```
aov(formula = disp ~ cyl, data = mtcars)
## Call:
## aov(formula = disp ~ cyl, data = mtcars)
##
## Terms:
## cyl Residuals
## Sum of Squares 387454.1 88730.7
## Deg. of Freedom 1 30
##
## Residual standard error: 54.38465
## Estimated effects may be unbalanced
summary(aov(formula = disp ~ cyl, data = mtcars))
## Df Sum Sq Mean Sq F value Pr(>F)
## cyl 1 387454 387454 131 1.8e-12 ***
## Residuals 30 88731 2958
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Not very likely.

## Version

This document was created using:

```
version
##
## platform x86_64-w64-mingw32
## arch x86_64
## os mingw32
```

```
## system x86_64, mingw32
## status
## major 4
## minor 0.0
## year 2020
## month 04
## day 24
## svn rev 78286
## language R
## version.string R version 4.0.0 (2020-04-24)
## nickname Arbor Day
```

